A square complex $n \times n$ matrix Z has n right eigenvectors and n left eigenvectors:

$$Z|i\rangle = z_i |i\rangle \qquad i = 1 - n$$

$$\langle j| Z = z_j \langle j| \qquad j = 1 - n$$

and n eigenvalues z_i, w_j.

"Proof" for $n = 2$.

$$Z = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

The eigenvalues z_i are the roots of the determinental equation

$$0 = \det(Z - zI) = \begin{vmatrix} a-z & b \\ c & d-z \end{vmatrix}$$

$$0 = (a-z)(d-z) - bc = z^2 - z(a+d) + ad - bc$$

So the complex roots are

$$z_\pm = \frac{a+d \pm \sqrt{(a+d)^2 + 4(bc-ad)}}{2}.$$

Example $n=2$; Take $d=-a$. Then

$$z_\pm = \pm\sqrt{bc-ad} = \pm\sqrt{bc+a^2}$$

Does this work?

$$Z\begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{bc+a^2}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$\|$$

$$\begin{pmatrix} a & b \\ c & -a \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \sqrt{bc+a^2}\begin{pmatrix} x \\ y \end{pmatrix}$$

$$ax+by = \sqrt{bc+a^2}\,x \equiv rx$$

$$cx-ay = \sqrt{bc+a^2}\,y \equiv ry$$

So the first equation gives

$y = (r-a)x/b$ and the second

$y = cx/(r+a)$. These are consistent because

$$bc = (r+a)(r-a) = r^2-a^2 = bc+a^2-a^2 = bc.$$

So

$$|z_+\rangle = \begin{pmatrix} 1 \\ \frac{r-a}{b} \end{pmatrix} x.$$

From the left now:

$$(x, y)\begin{pmatrix} a & b \\ c & -a \end{pmatrix} = \sqrt{bc+a^2}\,(x, y)$$

$$(xa+yc,\ xb-ay) = r\,(x, y)$$

$$xa+yc = rx \qquad xb-ay = ry$$

$$y = \frac{r-a}{c}x \qquad y = \frac{xb}{r+a}$$

Consistency follows from

$$bc = (r-a)(r+a) = r^2-a^2 = bc+a^2-a^2 = bc.$$

For the big picture, google "SVD wiki."

Every $m \times n$ complex matrix M is given by

$$M = U\Sigma V^\dagger$$

where the $m\times m$ matrix U and the $n\times n$ matrix V are unitary ($U^\dagger U = I$ & $V^\dagger V = I$), and the $m\times n$ rectangular matrix Σ has nonnegative numbers on its diagonal and zeros elsewhere.