Classical Charged Currents Make Coherent States

The interaction of a classical (aka, external, c-number, prescribed) current $\vec{j}(x,t)$ with the electromagnetic field $\vec{A}(x,t)$ is

$$V_I(t) = -\frac{1}{c}\int d^3x\ j(x,t)\cdot A(x,t) \qquad 1$$

in the interaction picture. If the current is turned on at time $t=0$, and the state of the field $\vec{A}$ is the vacuum $|0\rangle$ at $t=0$, then at time t $|0\rangle$ will evolve to

$$|0,t\rangle_I = T\left\{ e^{+\frac{i}{\hbar c}\int d^3x \int_0^t \vec{j}(x,t')\cdot\vec{A}(x,t')dt'}\right\}|0\rangle. \qquad (2)$$

Repeated use of the identity

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]}, \qquad (3)$$

which holds when $[A,B]$ commutes with A and B, gives

$$|0,t\rangle_I = e^{\frac{i}{\hbar c}\int d^3x \int_0^t \vec{A}(x,t')\cdot\vec{j}(x,t')dt' + i\varphi}|0\rangle \qquad (4)$$

in which φ is a real and irrelevant overall phase.

The state $|0,t\rangle_I$ is the coherent state

$$|0,t\rangle_I = \exp\Big(\sum_{kn} \alpha(k,n) a^\dagger(k,n) - \alpha^*(k,n) a(k,n) \Big) |0\rangle e^{i\varphi} \tag{5}$$

of amplitude

$$\alpha(k,n) = \frac{i}{\hbar c} \int d^3x \int_0^t dt' \left(\frac{hc^2}{\omega_k V} \right) j(x,t') \cdot e^*(k,n) e^{-ik\cdot x + i\omega_k t'} \tag{6}$$

which is a kind of Fourier transform of the current $\vec{j}(x,t')$.

The mean value of the field $\vec{A}(x,t)$ in the coherent state

$$|\{\alpha_i\}\rangle = |0,t\rangle_I \tag{7}$$

is

$$\langle \{\alpha\} | A(x,t) | \{\alpha\} \rangle = \sum_{kn} \left(\frac{hc^2}{\omega V} \right)^{\frac{1}{2}} \Big[e(k,n) e^{ikx - i\omega t} \alpha(k,n) + e^*(k,n) e^{-ikx + i\omega t} \alpha^*(k,n) \Big] \tag{8}$$

because

$$a(k,n) | \{\alpha\} \rangle = \alpha(k,n) | \{\alpha\} \rangle. \tag{9}$$

All this was worked out by Glauber in Phys. Rev. 84(3), 395 (1951): classical currents make quantum fields that are coherent and so most classical.