$$|x\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |y\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|R\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |L\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$|\theta\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

Note that $|R\rangle$ and $|L\rangle$ photons pass thru both x & y polarizers with probability

$$|\langle x|R\rangle|^2 = \left|(1,0)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}\right|^2 = \frac{1}{2}|1|^2 = \frac{1}{2}$$

and

$$|\langle y|L\rangle|^2 = \left|(0,1)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}\right|^2 = \frac{1}{2}|-i|^2 = \frac{1}{2}$$

etc. In fact, they pass thru θ filters

$$|\langle \theta|R\rangle|^2 = \left|(\cos\theta, \sin\theta)\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}\right|^2$$

$$= \frac{1}{2}|\cos\theta + i\sin\theta|^2 = \frac{1}{2}$$

$$|\langle\theta|L\rangle|^2 = \frac{1}{2}|\cos\theta - i\sin\theta|^2 = \frac{1}{2}$$

with probability $\frac{1}{2}$.

Note that no real linear combination $|\psi\rangle$ of $|x\rangle$ & $|y\rangle$ could do this; Let

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} \qquad a, b \text{ real.}$$

Then since $\langle\psi|\psi\rangle = 1 = a^2 + b^2$, we may set $a = \cos\alpha$, $b = \sin\alpha$.

Then

$$|\langle\theta|\psi\rangle|^2 = \left|(\cos\theta, \sin\theta)\begin{pmatrix}\cos\alpha \\ \sin\alpha\end{pmatrix}\right|^2$$

$$= |\cos\theta\cos\alpha + \sin\theta\sin\alpha|^2 = \cos^2(\theta-\alpha).$$

So any real linear combination of $|x\rangle$ & $|y\rangle$ passes thru filter at θ with probability $\cos^2(\theta-\alpha)$ which is $\frac{1}{2}$ only when $\theta - \alpha = \frac{\pi}{4} + n\pi$.