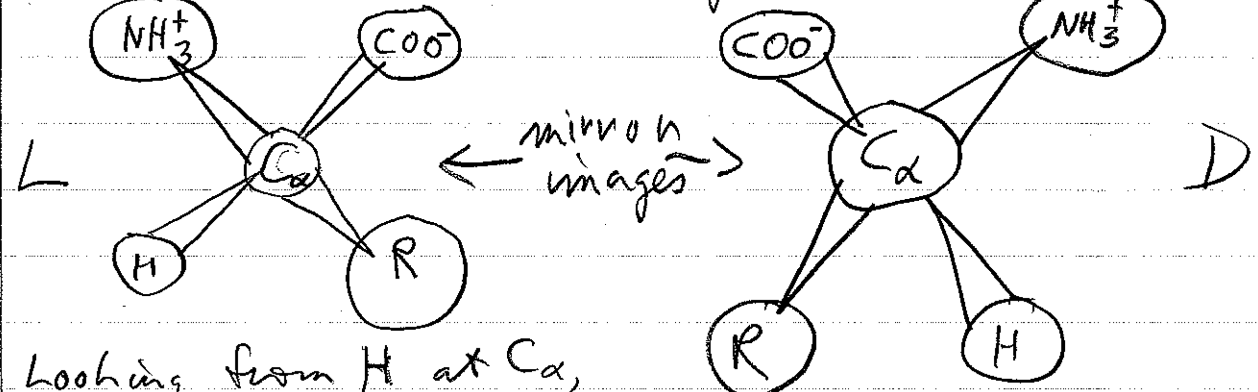
Chiral Molecules and Polarized Light

Many of the molecules in living things occur in one of two forms that differ by the reflection of one or all three spatial axes.

Proteins consist of L-amino acids



Looking from H at C_α, we see CORN in a clockwise sense.

Now CORN occurs in a CCW sense.

Molecules that look different in mirrors are chiral.

A solution of chiral molecules has different indices of refraction for left and right circularly polarized light. Call these n_R and n_L, so that

$$k_R = \frac{\omega}{c} n_R \quad \text{and} \quad k_L = \frac{\omega}{c} n_L .$$

$$v_R = \frac{c}{n_R} \qquad v_L = \frac{c}{n_L}$$

Recall

$$|R\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ i\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|x\rangle + i|y\rangle\right)$$

$$|L\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\ -i\end{pmatrix} = \frac{1}{\sqrt{2}}\left(|x\rangle - i|y\rangle\right).$$

Suppose the initial photons are polarized in the x-direction. So the initial state is

$$|in\rangle = \frac{1}{\sqrt{2}}\left(|R\rangle + |L\rangle\right) = |x\rangle$$

The photons are going in the z-direction, so the amplitude to get $|x\rangle$ at z,t is

$$\langle z,t,x|in\rangle = \frac{1}{\sqrt{2}}\left(\langle z,t,x|R\rangle + \langle z,t,x|L\rangle\right)$$

$$= \frac{1}{2\sqrt{h}}\left(e^{i(k_R z - \omega t)} + e^{i(k_L z - \omega t)}\right)$$

$$= \frac{1}{2\sqrt{h}}\left(e^{i\omega\left(\frac{n_R z}{c} - t\right)} + e^{i\omega\left(\frac{n_L z}{c} - t\right)}\right)$$

$$= \frac{e^{-i\omega t}}{2\sqrt{h}}\left(e^{i\omega n_R z/c} + e^{i\omega n_L z/c}\right)$$

or

$$\langle z,t|X\rangle = \frac{e^{-i\omega t}\, e^{i\omega\frac{(n_R+n_L)z}{2c}}}{2\sqrt{h}} \left(e^{i\frac{\omega}{c}(n_R-n_L)z} + e^{-i\frac{\omega}{c}(n_R-n_L)z} \right)$$

$$= \frac{1}{\sqrt{h}}\, e^{-i\omega t + i\omega\frac{(n_R+n_L)z}{2c}} \cos\left[\frac{\omega}{c}(n_R-n_L)z\right]$$

And the amplitude to find $|y\rangle$ at z,t is

$$\langle z,t,y|in\rangle = \frac{1}{\sqrt{2}}\left(\langle z,t,y|R\rangle + \langle z,t,y|L\rangle\right)$$

$$= \frac{1}{2}\left(i\, e^{i(k_R z-\omega t)} - i\, e^{i(k_L z-\omega t)} \right)$$

$$= \frac{e^{-i\omega t + i\omega\frac{(n_R+n_L)z}{2c}}}{2\sqrt{h}} \left(i\, e^{i\frac{\omega}{c}(n_R-n_L)z} - i\, e^{-i\frac{\omega}{c}(n_R-n_L)z} \right)$$

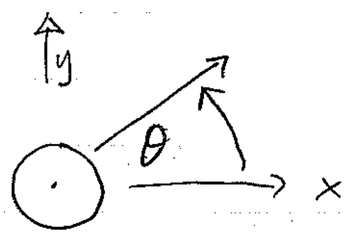
$$= -\frac{e^{-i\omega t + i\omega\frac{(n_R+n_L)z}{2c}}}{\sqrt{h}} \sin\left[\frac{\omega}{c}(n_R-n_L)z\right].$$

So apart from an over-all average phase the out state is

$$|out\rangle = \begin{pmatrix} \cos \frac{\omega}{c}(n_L - n_R)z \\ \sin \frac{\omega}{c}(n_L - n_R)z \end{pmatrix}.$$

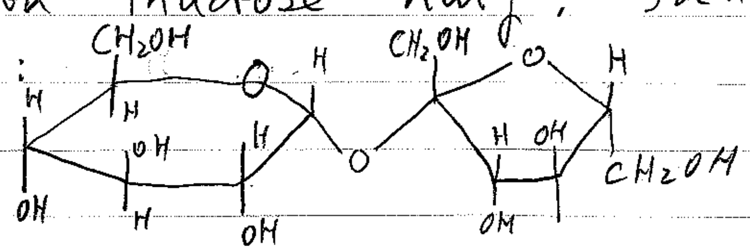
This is a state that still is linearly polarized, but whose plane of polarization has been rotated by the angle

$$\theta = \frac{\omega}{c}(n_L - n_R)z.$$

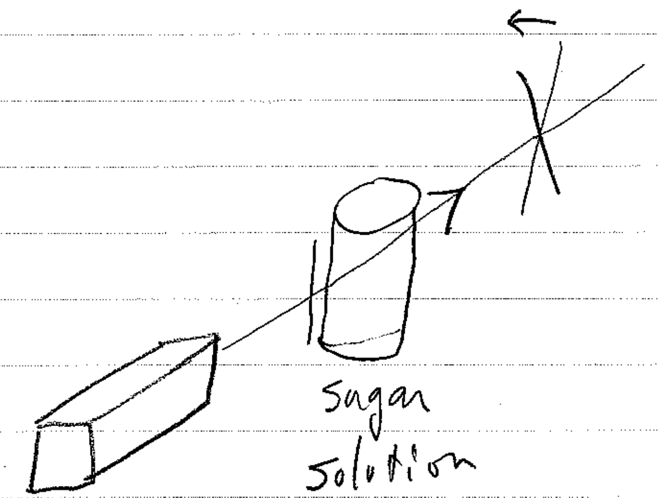


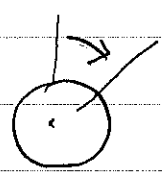
This is a right-handed rotation about the z-direction (of motion) by angle θ.

Sucrose is a disaccharide consisting of a six-carbon glucose ring connected, via an oxygen atom, to a five-carbon fructose ring. Sucrose is $C_{12}H_{22}O_{11}$:



A sucrose solution of 1 g per 100 ml rotates the plane of polarization by +66.47° in 10 cm. In the demo, however, the plane of polarization was rotated clockwise about the direction of motion of the photons. That would be a left-handed rotation. Presumably, a right-handed rotation is defined by chemists as a clock-wise rotation





when viewed while looking (dangerously) into the beam, as in the drawings.