Alternatively, we might simply examine the continuous time trajectory in y. For example, Figure 1.17 shows a result for an experiment involving chemical reactions (cf. Section 2.4.3). The vertical axis is the measured concentration \( g(t) \) of one chemical constituent at time \( t \) and the horizontal axis is the same quantity evaluated at \( t - (8.8 \text{ seconds}) \). We see that the delay coordinates \( y = (g(t), g(t - 8.8)) \) traces out a closed curve indicating a limit cycle.

**Problems**

1. Consider the following systems and specify (i) whether chaos can or cannot be ruled out for these systems, and (ii) whether the system is conservative or dissipative. Justify your answer

   \( \theta_{n+1} = [\theta_n + \Omega + 1.5 \sin \theta_n] \mod 2\pi, \)

   \( \theta_{n+1} = [\theta_n + \Omega + 0.5 \sin \theta_n] \mod 2\pi, \)

   \( x_{n+1} = [2x_n - x_{n-1} + k \sin x_n] \mod 2\pi, \)

   \( y_{n+1} = y_n + k(x_n - y_n)^2, \quad y_{n+1} = y_n + k(x_n - y_n)^2, \)

   \( dx/dt = v, \quad dv/dt = -wx + C \sin (\omega t - kx), \)

   \( dy/dt = C \cos z + A \sin x, \)

   \( dz/dt = A \cos x + B \sin y. \)

2. Consider the one-dimensional motion of a free particle which bounces elastically between a stationary wall located at \( x = 0 \) and a wall whose position oscillates with time and is given by \( x = L + \Delta \sin (\omega t) \). Derive a map relating the times \( T_n \) of the \( n \)th bounce off the oscillating wall and the particle speed \( v_n \) between the \( n \)th bounce and the \( (n+1) \)th bounce off the oscillating wall to \( T_{n+1} \) and \( v_{n+1} \). Assume that \( L \gg \Delta \) so that \( v_n(T_{n+1} - T_n) \approx 2L \). Is the map relating \( (T_n, v_n) \) to \( (T_{n+1}, v_{n+1}) \) conservative? Show that a new variable can be introduced in place of \( T_n \), such that the new variable is bounded and results in a map which yields the same \( v_n \) as for the original map for all \( n \).

3. Write a computer program to take iterates of the Hénon map. Considering the case \( A = 1.4, B = 0.3 \) and starting from an initial condition \( (x_0, y_0) = (0, 0) \) iterate the map 20 times and then plot the next 1000 iterates to get a picture of the attractor.

4. Plot the first 25 iterates of the map given by Eq. (1.8) starting from \( x_0 = 1/2; (a) \) for \( r = 3.8 \) (chaotic attractor), \( (b) \) for \( r = 2.5 \) (period one attractor), and \( (c) \) for \( r = 3.1 \) (period two attractor).

5. For the map (1.8) with \( r = 3.8 \) plot the iterates of the two orbits originating from the initial conditions \( x_0 = 0.2 \) and \( x_0 = 0.2 + 10^{-5} \) versus iterate number. When does the separation between the two orbits first exceed 0.2?