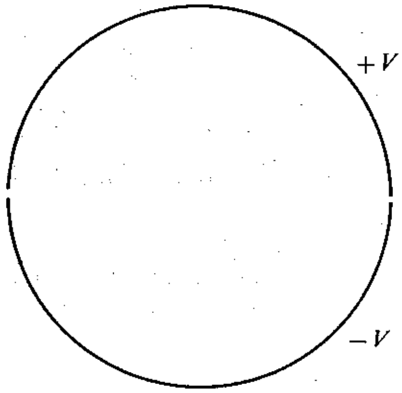

FIG. 14.8

Note. This variable width square wave is of some importance in electronic music.

14.3.6 A metal cylindrical tube of radius a is split lengthwise into two nontouching halves. The top half is maintained at a potential $+V$, the bottom half at a potential $-V$ (Fig. 14.8). Separate the variables in Laplace's equation and solve for the electrostatic potential for $r \le a$. Observe the resemblance between your solution for $r = a$ and the Fourier series for a square wave.

14.3.7 A metal cylinder is placed in a (previously) uniform electric field, E_0, the axis of the cylinder perpendicular to that of the original field.
(a) Find the perturbed electrostatic potential.
(b) Find the induced surface charge on the cylinder as a function of angular position.

14.3.8 Transform the Fourier expansion of a square wave, Eq. 14.3.6, into a power series. Show that the coefficients of x^1 form a *divergent* series. Repeat for the coefficients of x^3.

A power series cannot handle a discontinuity. These infinite coefficients are the result of attempting to beat this basic limitation on power series.

14.3.9 (a) Show that the Fourier expansion of $\cos ax$ is

$$\cos ax = \frac{2a\sin a\pi}{\pi}\left\{\frac{1}{2a^2} - \frac{\cos x}{a^2 - 1^2} + \frac{\cos 2x}{a^2 - 2^2} - \cdots\right\},$$

$$a_n = (-1)^n \frac{2a\sin a\pi}{\pi(a^2 - n^2)}.$$

(b) From the preceding result show that

$$a\pi \cot a\pi = 1 - 2\sum_{p=1}^{\infty} \zeta(2p)a^{2p}.$$

This provides an alternate derivation of the relation between the Riemann zeta function and the Bernoulli numbers, Eq. 5.151.

14.3.10 Derive the Fourier series expansion of the Dirac delta function $\delta(x)$ in the interval $-\pi < x < \pi$.
(a) What significance can be attached to the constant term?
(b) In what region is this representation valid?
(c) With the identity

(b) The new boundary conditions to be satisfied at $r = a$ are

$$\psi_{\text{in}}(a, \theta, \varphi) = \psi_{\text{out}}(a, \theta, \varphi)$$

$$\frac{\partial}{\partial r}\psi_{\text{in}}(a, \theta, \varphi) = \frac{\partial}{\partial r}\psi_{\text{out}}(a, \theta, \varphi)$$

or

$$\frac{1}{\psi_{\text{in}}}\frac{\partial \psi_{\text{in}}}{\partial r}\bigg|_{r=a} = \frac{1}{\psi_{\text{out}}}\frac{\partial \psi_{\text{out}}}{\partial r}\bigg|_{r=a}.$$

For $l = 0$ show that the boundary condition at $r = a$ leads to

$$f(E) = k\left\{\cot ka - \frac{1}{ka}\right\} + k'\left\{1 + \frac{1}{k'a}\right\}$$
$$= 0,$$

where $k = \sqrt{2ME}/\hbar$ and $k' = \sqrt{2M(V_0 - E)}/\hbar$.

(c) With $a = 1\hbar^2/Me^2$ (Bohr radius) and $V_0 = 4Me^4/2\hbar^2$, compute the possible bound states, $(0 < E < V_0)$.
Hint. Call a root-finding subroutine after you know the approximate location of the roots of

$$f(E), \quad (0, V_0).$$

(d) Show that when $a = 1\hbar^2/Me^2$ the minimum value of V_0 for which a bound state exists is $V_0 = 2.4674Me^4/2\hbar^2$.

11.7.27 In some nuclear stripping reactions the differential cross section is proportional to $(j_l(x))^2$, where l is the angular momentum. The location of the maximum on the curve of experimental data permits a determination of l, if the location of the (first) maximum of $j_l(x)$ is known. Compute the location of the first maximum of $j_1(x)$, $j_2(x)$, and $j_3(x)$.
Note. For better accuracy look for the first zero of $j_l'(x)$. Why is this more accurate than direct location of the maximum?

REFERENCES

McBride, E. B., *Obtaining Generating Functions*. New York: Springer-Verlag (1971).
An introduction to methods of obtaining generating functions.

Watson, G. N., *A Treatise on the Theory of Bessel Functions*, 2nd ed. Cambridge: Cambridge University Press (1952).
This is the definitive text on Bessel functions and their properties. Although difficult reading, it is invaluable as the ultimate reference.

Watson, G. N., *Theory of Bessel Functions*. Cambridge: Cambridge University Press.
See also the references listed at the end of Chapter 13.