A convenient choice of Dirac matrices, used by Weinberg, is
\[
\gamma = -i \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}, \quad \text{and} \quad \gamma^0 = -i \beta = -i \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}. \tag{1}
\]
They satisfy the anti-commutation relations
\[
[\gamma^a, \gamma^b]_+ = 2 \eta^{ab}, \tag{2}
\]
in which the flat space-time metric is
\[
\eta^{ab} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{3}
\]
Under hermitian conjugation, they transform as \(\gamma^\dagger = \gamma\) and \((\gamma^0)^\dagger = -\gamma^0\).

For this choice of Dirac matrices, we may define Majorana and Dirac fields in terms of the scalar-like lawn
\[
\phi(x) = \int \frac{d^3p}{2\sqrt{(2\pi)^3p^0(p^0 + m)}} \left[ \begin{pmatrix} I \\ I \end{pmatrix} A(p)e^{ipx} + i \begin{pmatrix} \sigma_2 \\ -\sigma_2 \end{pmatrix} A^*(p)e^{-ipx} \right], \tag{4}
\]
where \(I\) and \(\sigma_2\) are the 2 \times 2 matrices
\[
I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{5}
\]
\(A(p)\) and \(A^*(p)\) are the 2-vectors
\[
A(p) = \begin{pmatrix} a(p, +) \\ a(p, -) \end{pmatrix} \quad \text{and} \quad A^*(p) = \begin{pmatrix} a^*(p, +) \\ a^*(p, -) \end{pmatrix}. \tag{6}
\]
\(p^0 = \sqrt{m^2 + p^2}\), and \(h = c = 1\). The lawn \(\phi(x)\) describes a single spin-one-half particle that is its own anti-particle.

Since \(m^2 + p^2 = m^2 + p^2 - (p^0)^2 = 0\), the lawn \(\phi(x)\) satisfies the Klein-Gordon equation
\[
(m^2 + \partial_0^2 - \nabla^2)\phi(x) = (m^2 - \eta^{ab}\partial_a\partial_b)\phi(x) = 0. \tag{7}
\]
The Majorana field $\chi(x)$ is obtained from derivatives of the lawn $\phi(x)$:

$$\chi(x) = (m - \gamma^a \partial_a) \beta \phi(x).$$

(8)

It automatically satisfies the Dirac equation:

$$(\gamma^a \partial_a + m) \chi(x) = (m^2 - \gamma^a \gamma^b \partial_a \partial_b) \beta \phi(x) = 0.$$  

(12)

Suppose that there are two spin-one-half particles of the same mass $m$ described by the two operators $a_1(p, \sigma)$ and $a_2(p, \sigma)$ which satisfy the anticommutation relations

$$[a_i(p, \sigma), a_j^\dagger(p', \sigma')]_+ = \delta_{\sigma\sigma'} \delta^3(p - p').$$  

(9)

Then by following Eqs.(4–9) and defining two 2-vectors $A_i(p, \sigma)$ as in (6), we may construct the two lawns

$$\phi_i(x) = \int \frac{d^3p}{2\sqrt{(2\pi)^3p^0(p^0 + m)}} \left[ \begin{pmatrix} I \\ I \end{pmatrix} A_i(p)e^{ipx} + i \begin{pmatrix} \sigma_2 \\ -\sigma_2 \end{pmatrix} A_i^*(p)e^{-ipx} \right]$$  

(10)

and from them the two Majorana fields

$$\chi_i(x) = (m - \gamma^a \partial_a) \beta \phi_i(x)$$  

(11)

which satisfy the Dirac equation

$$\chi_i(x) = \gamma^a \partial_a + m \chi_i(x) = 0.$$  

(12)

But because the two lawns $\phi_i(x)$ are of the same mass, we may combine them into the complex lawn

$$\Phi(x) = \frac{1}{\sqrt{2}} [\phi_1(x) + i\phi_2(x)].$$  

(13)

From the complex operators

$$a(p, \sigma) = \frac{1}{\sqrt{2}} [a_1(p, \sigma) + ia_2(p, \sigma)]$$  

(14)
and
\[ a^c(p, \sigma) = \frac{1}{\sqrt{2}} [a_1(p, \sigma) - ia_2(p, \sigma)], \tag{15} \]
we may form the complex 2-vectors
\[ A(p) = \frac{1}{\sqrt{2}} [A_1(p) + iA_2(p)] = \left( \begin{array}{c} a(p, +) \\ a(p, -) \end{array} \right) \tag{16} \]
and
\[ A^c(p) = \frac{1}{\sqrt{2}} [A_1(p) - iA_2(p)] = \left( \begin{array}{c} a^c(p, +) \\ a^c(p, -) \end{array} \right). \tag{17} \]
The complex lawn involves \( A(p) \) and
\[ A^c(p) = \frac{1}{\sqrt{2}} [A_1(p) - iA_2(p)]^* = \frac{1}{\sqrt{2}} [A_1^*(p) + iA_2^*(p)] = \left( \begin{array}{c} a^{c*}(p, +) \\ a^{c*}(p, -) \end{array} \right), \tag{18} \]
in the form
\[ \Phi(x) = \int \frac{d^3p}{(2\pi)^3p^0(p^0 + m)} \left[ \left( \begin{array}{c} I \\ I \end{array} \right) A(p)e^{ipx} + i \left( \begin{array}{c} \sigma_2 \\ -\sigma_2 \end{array} \right) A^{c*}(p)e^{-ipx} \right]. \tag{19} \]

The Dirac field is then
\[ \psi(x) = (m - \gamma^a \partial_a) \beta \Phi(x) = (m - \gamma^a \partial_a) \beta \frac{1}{\sqrt{2}} [\phi_1(x) + i\phi_2(x)] = \frac{1}{\sqrt{2}} [\chi_1(x) + i\chi_2(x)]. \]
It satisfies the Dirac equation
\[ (\gamma^a \partial_a + m) \psi(x) = 0 \tag{20} \]
because the Majorana fields \( \chi_1 \) and \( \chi_2 \) do.

We have defined Majorana and Dirac fields in terms of Weinberg's choice of Dirac matrices. If one uses a different set of Dirac matrices
\[ \gamma'^a = S\gamma^a S^{-1}, \quad \beta' = S\beta S^{-1}, \tag{21} \]
then the fields and lawns should be multiplied from the left by the non-singular matrix \( S \):
\[ \Phi'(x) = S\Phi(x), \quad \psi'(x) = \psi(x), \quad etc. \tag{22} \]