3.3.12 A is 2×2 and orthogonal. Find the most general form of

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

Compare with two-dimensional rotation.

3.3.12 $1 = AA^T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & c \\ b & d \end{pmatrix}$, so

$1 = a^2 + b^2$ $\qquad$ $1 = c^2 + d^2$

$0 = ca + db$

We may satisfy $1 = a^2 + b^2$ and $1 = c^2 + d^2$ by setting $a = \cos\theta$, $b = \sin\theta$, $c = \sin\phi$, and $d = \cos\phi$. Now the $\perp$ condition is

$0 = \cos\theta \sin\phi + \sin\theta\cos\phi = \sin(\theta + \phi)$.

So we set $\phi = -\theta + n\pi$, which give

$A(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ for $n = 0$

and $A(-\theta)$ for $n = 1$.

3.4.7 **A and B are two noncommuting Hermitian matrices:**

$$AB - BA = iC.$$

Prove that C is Hermitian.

3.4.7 $C = -i(AB - BA)$

$C^\dagger = i(B^\dagger A^\dagger - A^\dagger B^\dagger)$

$= i(BA - AB) = -i(AB -$

$= -i(AB - BA)$

$= C.$

3.4.9 Two matrices A and B are each Hermitian. Find a necessary and sufficient condition for their product AB to be Hermitian.

ANS. $[A, B] = 0$.

3.4.9 $(AB)^\dagger = B^\dagger A^\dagger = BA = AB$

5. $[A, B] = 0$.

3.6.20 Two equal masses are connected to each other and to walls by springs as shown in Fig. 3.8. The masses are constrained to stay on a horizontal line.

(a) Set up the Newtonian acceleration equation for each mass.
(b) Solve the secular equation for the eigenvectors.
(c) Determine the eigenvectors and thus the normal modes of motion.

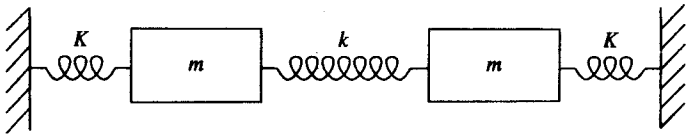


FIGURE 3.8 Oscillator.

3.6.20 a) $m\ddot{x} = k(y-x) - Kx$ $\quad$ (let $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} e^{i\omega t}$.

$m\ddot{y} = -Ky + k(x-y)$

(b) $\begin{pmatrix} -k-k & k \\ k & -k-k \end{pmatrix}\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = -m\omega^2 \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$, so

$$0 = \begin{vmatrix} m\omega^2 - K - k & k \\ k & m\omega^2 - K - k \end{vmatrix} = (m\omega^2 - K - k)^2 - k^2$$

$m\omega^2 - K - k = \pm k$ $\qquad$ $m\omega^2 = K + 2k$

$m\omega^2 = K$

If $m\omega^2 = K + 2k$, then

$-(K+k)x^0 + k y^0 = -(K+2k)x_0$ or $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = x_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

If $m\omega^2 = K$, then

$-(K+k)x_0 + k y_0 = -K x_0$ or $\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = x_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(c) So the normal modes are

$\begin{pmatrix} x \\ y \end{pmatrix} = x_0 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{\pm i t\sqrt{K/m}}$ and $\begin{pmatrix} x \\ y \end{pmatrix} = x_0 \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{\pm i t\sqrt{(K+2k)/m}}$

3.6.20 (c)(2) There are really four solutions here since the ω's can be $\pm$ the square roots. So one mode is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \left[a \cos t\sqrt{\frac{k}{m}} + b \sin t\sqrt{\frac{k}{m}} \right]$$

and the other is

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \left[c \cos t\sqrt{(K+2k)/m} + d \sin t\sqrt{(K+2k)/m} \right].$$

4.1.1 Show that an $n \times n$ orthogonal matrix has $n(n-1)/2$ independent parameters.
Hint. The orthogonality condition, Eq. (3.71), provides constraints.

4.1.1 $1 = A A^T$ implies the n^2 constraints

$$\delta_{ij} = a_{ik} a_{jk}$$

of which only the n for $i = j$ and the $n(n-1)/2$ for $i \neq j$ are independent. So the number of constraints is

$$N_C = n + n\frac{(n-1)}{2} = n\left(1 + \frac{n-1}{2}\right) = \frac{n(n+1)}{2}.$$

So the number of independent parameters is

$$n^2 - \frac{n(n+1)}{2} = \frac{n^2}{2} - n = \frac{n(n-1)}{2}.$$

4.1.3 The special linear group SL(2) consists of all 2×2 matrices (with complex elements) having a determinant of $+1$. Show that such matrices form a group. *Note.* The SL(2) group can be related to the full Lorentz group in Section 4.4, much as the SU(2) group is related to SO(3).

4.1.3 If D and D' are in $SL(2, \mathbb{C})$, then

$\det(DD') = \det D \det D' = 1 \cdot 1 = 1, \neq 0$

multiplication is closed.

$\det D^{-1} = 1/\det D = 1/1 = 1.$

each matrix has an inverse in $SL(2, \mathbb{C})$

$\det 1 = 1$

the identity matrix is in $SL(2, \mathbb{C})$.

Matrix multiplication is associative.

4.3.1 Show that (a) $[J_+, J^2] = 0$, (b) $[J_-, J^2] = 0$.

4.3.1 a) $[J_+, J^2] = [J_1 + iJ_2, J^2]$

$= [J_1, J^2] + i[J_2, J^2] = 0 + i \cdot 0 = 0$.

b) $[J_-, J^2] = [J_1, J^2] - i[J_2, J^2] = 0$.

This is an extra-credit problem.

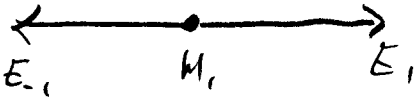
4.3.2 Derive the root diagram of SU(3) in Fig. 4.6b from the generators λ_i in Eq. (4.61).
Hint. Work out first the SU(2) case in Fig. 4.6a from the Pauli matrices.

4.3.2 First SU(2). $H_1 = \frac{\sigma_3}{2}$, $E_{\pm 1} = \frac{\sigma_1 \pm i\sigma_2}{2}$

$$[H_1, E_{\pm 1}] = \left[\frac{\sigma_3}{2}, \frac{\sigma_1 \pm i\sigma_2}{2}\right] = \left(i\epsilon_{312}\frac{\sigma_2}{2} \mp \epsilon_{321}\frac{\sigma_1}{2}\right)$$

$$= \pm\frac{\sigma_1}{2} + i\frac{\sigma_2}{2} = \pm\left(\frac{\sigma_1 \pm i\sigma_2}{2}\right)$$

$$= \pm E_{\pm 1}.$$



So $\alpha_\pm = \pm 1$.

Now SU(3): $H_1 = \frac{\lambda_3}{2}$, $H_2 = \frac{\lambda_8}{2}$. Let's first find the ev's and weights of the defining rep.

$$\frac{\lambda_3}{2}\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\\0\end{pmatrix} \qquad \frac{\lambda_8}{2}\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{\sqrt{3}}{6}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

So $\mu_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$. Next

$$\frac{\lambda_3}{2}\begin{pmatrix}0\\1\\0\end{pmatrix} = -\frac{1}{2}\begin{pmatrix}0\\1\\0\end{pmatrix} \qquad \frac{\lambda_8}{2}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{\sqrt{3}}{6}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

So $\mu_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$.

$$\frac{\lambda_3}{2}\begin{pmatrix}0\\0\\1\end{pmatrix} = 0 \qquad \frac{\lambda_8}{2}\begin{pmatrix}0\\0\\1\end{pmatrix} = -\frac{\sqrt{3}}{3}\begin{pmatrix}0\\0\\1\end{pmatrix}.$$

So $\mu_3 = \left(0, -\sqrt{3}/3\right)$.

4.3.2(2) Now for any rep. D

$$H_i E_{\pm\alpha} |\mu, D\rangle = [H_i, E_{\pm\alpha}] |\mu, D\rangle + E_{\pm\alpha} H_i |\mu, D\rangle$$

$$= \pm \alpha_i E_{\pm\alpha} |\mu, D\rangle + \mu_i E_{\pm\alpha} |\mu, D\rangle$$

$$= (\mu_i \pm \alpha_i) E_{\pm\alpha} |\mu, D\rangle .$$

So the root vectors point from one $\vec{\mu}$ to another, e.g., $\mu' = \mu - \alpha$ or $\vec{\alpha} = \vec{\mu} - \vec{\mu}'$.
So

$$\alpha_{12} = \mu_1 - \mu_2 = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{6}\right) - \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{6}\right)$$

$$= (1, 0) \; ; \quad \alpha_{21} = -\alpha_{12} .$$

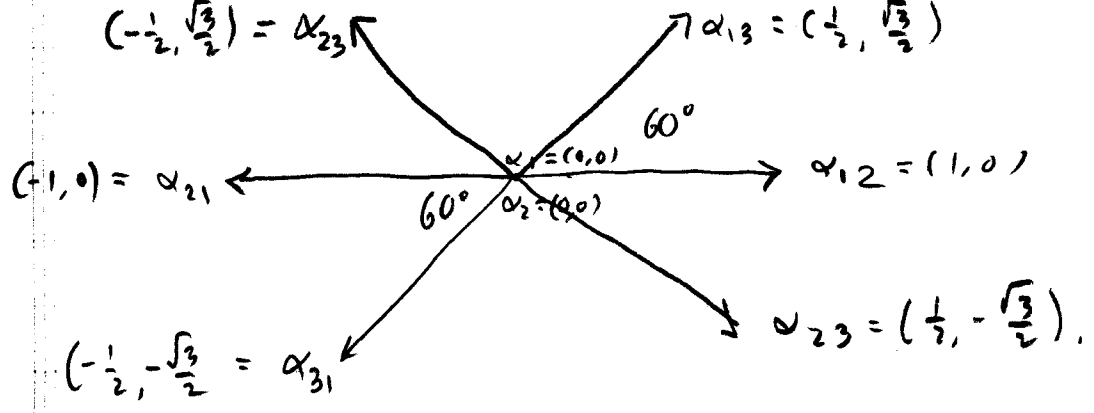
$$\alpha_{13} = \mu_1 - \mu_3 = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{6}\right) - \left(0, -\tfrac{\sqrt{3}}{3}\right) = \left(\tfrac{1}{2}, \tfrac{3\sqrt{3}}{6}\right) = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$$\alpha_{31} = -\alpha_{13} .$$

$$\alpha_{23} = \mu_2 - \mu_3 = \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{6}\right) - \left(0, -\tfrac{\sqrt{3}}{3}\right) = \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$

$$\alpha_{32} = -\alpha_{23} .$$

Also $\alpha_1 = (0,0)$ and $\alpha_2 = (0,0)$ are the root vectors of H_1 & H_2. So



Special problem one:

1. **Find the multiplication table for a group with three elements and prove that it is unique.**

Call the three elements e, a, and b, with e the identity. Then $ea = a$; $eb = b$; and $ee = e$. We must figure out what a^2, b^2, ab, and ba are.

If $ab = a$, then $a^{-1}(ab) = a^{-1}a = e = (a^{-1}a)b = eb = b$ so that we'd have $b = e$. Then the group would have two elements, not three. So $ab \neq a$.
Similarly, $ab = b \Rightarrow a = e$. So $ab \neq b$.
Thus $ab = e$.

Similarly, $ba = e$. So $b = a^{-1}$.

If $a^2 = a$, then $a = e$; so $a^2 \neq a$.
If $a^2 = e$, then $a = a^{-1} = b$. So $a^2 \neq e$
So $a^2 = b$.

If $b^2 = e$, then $abb = a$ or $b = a$.
If $b^2 = b$, then $b = e$; so $b^2 \neq b$.
So $b^2 = a$.

So the multiplication table is

	e	a	b
e	e	a	b
a	a	b	e
b	b	e	a

.

2. Consider the 2 x 3 matrix A with elements
(1 2 3)
(-3 0 1).
(A(1,1) = A(2,3) = 1, etc.) Perform the singular value decomposition
A = U S V'
where V' is the transpose of V. Find the singular values and the real matrices U and V.

CXML

We use the Lapack driver dgesvd: LAPACK version 3.0

dgesvd(3)

PURPOSE

DGESVD - compute the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors

SYNTAX

```
SUBROUTINE DGESVD( JOBU, JOBVT, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
                   LWORK, INFO )

    CHARACTER      JOBU, JOBVT

    INTEGER        INFO, LDA, LDU, LDVT, LWORK, M, N

    DOUBLE         PRECISION A( LDA, * ), S( * ), U( LDU, * ), VT( LDVT, *
                   ), WORK( * )
```

DESCRIPTION

DGESVD computes the singular value decomposition (SVD) of a real M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written

A = U * SIGMA * transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M orthogonal matrix, and V is an N-by-N orthogonal matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**T, not V.

ARGUMENTS

JOBU (input) CHARACTER*1
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A; = 'N': no columns of U (no left singular vectors) are computed.

JOBVT (input) CHARACTER*1
Specifies options for computing all or part of the matrix V**T:
= 'A': all N rows of V**T are returned in the array VT;
= 'S': the first min(m,n) rows of V**T (the right singular

vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**T (the right singular vectors) are overwritten on the array A; = 'N': no rows of V**T (no right singular vectors) are computed.

JOBVT and JOBU cannot both be 'O'.

M (input) INTEGER
The number of rows of the input matrix A. M >= 0.

N (input) INTEGER
The number of columns of the input matrix A. N >= 0.

A (input/output) DOUBLE PRECISION array, dimension (LDA,N)
On entry, the M-by-N matrix A. On exit, if JOBU = 'O', A is overwritten with the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBVT = 'O', A is overwritten with the first min(m,n) rows of V**T (the right singular vectors, stored rowwise); if JOBU .ne. 'O' and JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input) INTEGER
The leading dimension of the array A. LDA >= max(1,M).

S (output) DOUBLE PRECISION array, dimension (min(M,N))
The singular values of A, sorted so that S(i) >= S(i+1).

U (output) DOUBLE PRECISION array, dimension (LDU,UCOL)
(LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU = 'S'. If JOBU = 'A', U contains the M-by-M orthogonal matrix U; if JOBU = 'S', U contains the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBU = 'N' or 'O', U is not referenced.

LDU (input) INTEGER
The leading dimension of the array U. LDU >= 1; if JOBU = 'S' or 'A', LDU >= M.

VT (output) DOUBLE PRECISION array, dimension (LDVT,N)
If JOBVT = 'A', VT contains the N-by-N orthogonal matrix V**T; if JOBVT = 'S', VT contains the first min(m,n) rows of V**T (the right singular vectors, stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input) INTEGER
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace/output) DOUBLE PRECISION array, dimension (LWORK)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK; if INFO > 0, WORK(2:MIN(M,N)) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in S (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

LWORK (input) INTEGER
The dimension of the array WORK. LWORK >= 1. LWORK >= MAX(3*MIN(M,N)+MAX(M,N),5*MIN(M,N)). For good performance, LWORK should generally be larger.

If LWORK = -1, then a workspace query is assumed; the routine only calculates the optimal size of the WORK array, returns this value as the first entry of the WORK array, and no error message related to LWORK is issued by XERBLA.

INFO (output) INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if DBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of WORK above for details.

CXML Home Page

```
module defs
  integer, parameter :: i4b = selected_int_kind(9)
  integer, parameter :: i2b = selected_int_kind(4)
  integer, parameter :: ip = kind(1)
  integer, parameter :: sp = kind(1.0)
  integer, parameter :: dp = kind(1.0d0)
end module defs
```

```
program svd
  use defs; implicit none
  character(len=1),parameter::jobu='a',jobvt='a'
  integer(i4b)::info,i
  integer(i4b),parameter::m=2,n=3,lda=m,ldu=m,ldvt=n,&
       lwork=100
  real(dp),dimension(lda)::S
  real(dp),dimension(lwork)::work
  real(dp),dimension(lda,n)::A
  real(dp),dimension(m,m)::U
  real(dp),dimension(n,n)::VT
  do i = 1,3
     A(1,i) = real(i,dp)
  end do
  A(2,1) = -3.0; A(2,2) = 0.0; A(2,3) = 1.0
  call dgesvd(jobu,jobvt,m,n,A,lda,S,U,ldu,VT,ldvt,&
       work,lwork,info)
  open(7,file='problem-2')
  write(7,*)'The',lda,' singular values are:'
  do i = 1, lda
     write(7,*) 'S(',i,') =',S(i)
  end do
  write(7,*)'The',lda,' left singular vectors are:'
  write(7,*)
  do i = 1, lda
     write(7,*) 'L(',i,') =',U(:,i)
  end do
  write(7,*)
  write(7,*)'The',lda,' right singular vectors are:'
  do i = 1, lda
     write(7,*) 'R(',i,') =',VT(i,:)
  end do
end program svd
```

```
 The 2  singular values are:
 S( 1 ) = 3.741657386773942
 S( 2 ) = 3.162277660168379
 The 2  left singular vectors are:

 L( 1 ) = 1.000000000000000 0.000000000000000E+00
 L( 2 ) = 0.000000000000000E+00 1.000000000000000

 The 2  right singular vectors are:
 R( 1 ) = 0.2672612419124243 0.5345224838248487 0.8017837257372731
 R( 2 ) = -0.9486832980505137 0.000000000000000E+00 0.3162277660168381
```

3. Consider the 6 x 9 matrix A with elements
A(j,k) = x + x**j + i*(y - y**k)
where x = 1.1 and y = 1.02 and * means multiplication while ** means exponentiation. Find the singular values, and the first left and right singular vectors. **Lapack** is one way to do this problem. This assignment is due on the Tuesday after Thanksgiving.

Since A is complex, we use zgesvd instead of dgesvd.

ZGESVD(1) LAPACK driver routine (version 1.1) ZGESVD(1)

NAME

ZGESVD - compute the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors

SYNOPSIS

```
SUBROUTINE ZGESVD( JOBU, JOBVT, M, N, A, LDA, S, U, LDU, VT, LDVT, WORK,
                   LWORK, RWORK, INFO )

    CHARACTER      JOBU, JOBVT

    INTEGER        INFO, LDA, LDU, LDVT, LWORK, M, N

    DOUBLE         PRECISION RWORK( * ), S( * )

    COMPLEX*16     A( LDA, * ), U( LDU, * ), VT( LDVT, * ), WORK( * )
```

PURPOSE

ZGESVD computes the singular value decomposition (SVD) of a complex M-by-N matrix A, optionally computing the left and/or right singular vectors. The SVD is written

A = U * SIGMA * conjugate-transpose(V)

where SIGMA is an M-by-N matrix which is zero except for its min(m,n) diagonal elements, U is an M-by-M unitary matrix, and V is an N-by-N unitary matrix. The diagonal elements of SIGMA are the singular values of A; they are real and non-negative, and are returned in descending order. The first min(m,n) columns of U and V are the left and right singular vectors of A.

Note that the routine returns V**H, not V.

ARGUMENTS

JOBU (input) CHARACTER*1
Specifies options for computing all or part of the matrix U:
= 'A': all M columns of U are returned in array U:
= 'S': the first min(m,n) columns of U (the left singular vectors) are returned in the array U; = 'O': the first min(m,n) columns of U (the left singular vectors) are overwritten on the array A; = 'N': no columns of U (no left singular vectors) are computed.

JOBVT (input) CHARACTER*1
Specifies options for computing all or part of the matrix V**H:
= 'A': all N rows of V**H are returned in the array VT;
= 'S': the first min(m,n) rows of V**H (the right singular vectors) are returned in the array VT; = 'O': the first min(m,n) rows of V**H (the right singular vectors) are overwritten on the array A; = 'N': no rows of V**H (no right singular vectors) are computed.

JOBVT and JOBU cannot both be 'O'.

M (input) INTEGER
The number of rows of the input matrix A. M >= 0.

N (input) INTEGER
The number of columns of the input matrix A. N >= 0.

A (input/output) COMPLEX*16 array, dimension (LDA,N)
On entry, the M-by-N matrix A. On exit, if JOBU = 'O', A is overwritten with the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBVT = 'O', A is overwritten with the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBU .ne. 'O' and JOBVT .ne. 'O', the contents of A are destroyed.

LDA (input) INTEGER
The leading dimension of the array A. LDA >= max(1,M).

S (output) DOUBLE PRECISION array, dimension (min(M,N))
The singular values of A, sorted so that S(i) >= S(i+1).

U (output) COMPLEX*16 array, dimension (LDU,UCOL)
(LDU,M) if JOBU = 'A' or (LDU,min(M,N)) if JOBU = 'S'. If JOBU = 'A', U contains the M-by-M unitary matrix U; if JOBU = 'S', U contains the first min(m,n) columns of U (the left singular vectors, stored columnwise); if JOBU = 'N' or 'O', U is not referenced.

LDU (input) INTEGER
The leading dimension of the array U. LDU >= 1; if JOBU = 'S' or 'A', LDU >= M.

VT (output) COMPLEX*16 array, dimension (LDVT,N)
If JOBVT = 'A', VT contains the N-by-N unitary matrix V**H; if JOBVT = 'S', VT contains the first min(m,n) rows of V**H (the right singular vectors, stored rowwise); if JOBVT = 'N' or 'O', VT is not referenced.

LDVT (input) INTEGER
The leading dimension of the array VT. LDVT >= 1; if JOBVT = 'A', LDVT >= N; if JOBVT = 'S', LDVT >= min(M,N).

WORK (workspace/output) COMPLEX*16 array, dimension (LWORK)
On exit, if INFO = 0, WORK(1) returns the optimal LWORK.

LWORK (input) INTEGER
The dimension of the array WORK. LWORK >= 1. LWORK >= 2*MIN(M,N)+MAX(M,N). For good performance, LWORK should generally be larger.

RWORK (workspace) DOUBLE PRECISION array, dimension (5*max(M,N))
On exit, if INFO > 0, RWORK(1:MIN(M,N)-1) contains the unconverged superdiagonal elements of an upper bidiagonal matrix B whose diagonal is in S (not necessarily sorted). B satisfies A = U * B * VT, so it has the same singular values as A, and singular vectors related by U and VT.

INFO (output) INTEGER
= 0: successful exit.
< 0: if INFO = -i, the i-th argument had an illegal value.
> 0: if ZBDSQR did not converge, INFO specifies how many superdiagonals of an intermediate bidiagonal form B did not converge to zero. See the description of RWORK above for details.

Back to the listing of simple driver routines

```
program svd3
  use defs; implicit none
  character(len=1),parameter::jobu='a',jobvt='a'
  integer(i4b)::info,i,j
  integer(i4b),parameter::m=6,n=9,lda=m,ldu=m,ldvt=n,&
       lwork=200
  real(dp),dimension(lda)::S
  real(dp),dimension(5*n)::rwork
  complex(dp),dimension(lwork)::work
  complex(dp),dimension(lda,n)::A
  complex(dp),dimension(m,m)::U
  complex(dp),dimension(n,n)::VT
  real(dp),parameter::x=1.1,y=1.02
  do j = 1, n; do i = 1,m
     A(i,j) = cmplx(x + x**i,y - y**j)
  end do; end do
  call zgesvd(jobu,jobvt,m,n,A,lda,S,U,ldu,VT,ldvt,&
       work,lwork,rwork,info)
  open(7,file='problem-3')
  write(7,*)'The',lda,' singular values are:'
  do i = 1, lda
     write(7,*) 'S(',i,') =',S(i)
  end do
  write(7,*)
  write(7,*)'The first left singular vector is:'
  write(7,*) 'L(',1,') =',U(:,1)
  write(7,*)
  write(7,*)'The first right singular vector is:'
  write(7,*) 'R(',1,') =',conjg(VT(1,:))
end program svd3
```

```
 The 6  singular values are:
 S( 1 ) = 18.56999776537926
 S( 2 ) = 3.771581343789893E-02
 S( 3 ) = 2.146946191637828E-15
 S( 4 ) = 4.725174820839520E-16
 S( 5 ) = 3.158251745532310E-16
 S( 6 ) = 1.959748226820287E-16

 The first left singular vector is:
 L( 1 ) = (-0.3557955535806104,2.025284796465866E-23) (-0.3735483605589319,-6.900867214146
602E-04) (-0.3930763943657091,-1.449180020961566E-03) (-0.4145573085094160,-2.284185641904
907E-03) (-0.4381862640459296,-3.202689880505448E-03) (-0.4641781228317201,-4.213044842110
218E-03)

 The first right singular vector is:
 R( 1 ) = (-0.3330537031449926,-1.709902107912780E-03) (-0.3330406972638836,-4.38962933910
8539E-03) (-0.3330274312664829,-7.122950840891350E-03) (-0.3330138999467118,-9.91093927184
8393E-03) (-0.3330000980034902,-1.275468686462866E-02) (-0.3329860200217367,-1.56553093407
5532E-02) (-0.3329716604794932,-1.861394444257107E-02) (-0.3329570137479247,-2.16317519332
3814E-02) (-0.3329420740794449,-2.470991604349603E-02)
```